Computing Mitered Offset Curves Based on Straight Skeletons

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Offsetting of input polygon $P$ yields wavefront $WF(P, t)$ for offset distance $t$.

Wavefront propagation with unit speed via continued offsetting: shrinking process, where offset distance $t$ equals time.

Straight skeleton $SK(P)$ is union of traces of wavefront vertices.
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Straight Skeletons — Motivation


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**Aichholzer&Alberts&Aurenhammer&Gärtner (1995)**

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- Straight skeleton $\mathcal{SK}(\mathcal{P})$ is union of traces of wavefront vertices.
Change of Wavefront Topology

**Edge event**
- Wavefront topology changes over time.
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- *Edge event*: an edge of $\mathcal{WF}(P, t)$ vanishes.
Change of Wavefront Topology

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- Wavefront topology changes over time.
- *Edge event*: an edge of $WF(P, t)$ vanishes.
- Such a change of topology corresponds to a *node* of $SK(P)$.
Change of Wavefront Topology

**Split event**

- Wavefront topology changes over time.

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Change of Wavefront Topology

Split event

- Wavefront topology changes over time.
- *Split event*: wavefront splits into two parts.

Split event

Also split events correspond to nodes of \( \text{SK}(P) \).

split event
Change of Wavefront Topology

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- *Split event*: wavefront splits into two parts.
- Also split events correspond to *nodes* of $SK(P)$. 

![Diagram illustrating split event]
Change of Wavefront Topology

Split event

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- Also split events correspond to *nodes* of $SK(\mathcal{P})$. 

![Diagram showing wavefront topology changes and split events](image-url)
Definition

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**Basic facts**

- The topology of the wavefront $WF(P, t)$ changes with time/distance $t$ due to edge and split events.
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- No metric-based definition of straight skeletons exists.
- If $P$ has $n$ segments then $SK(P)$ consists of $O(n)$ nodes and $O(n)$ straight-line edges.
The definition of straight skeletons can be extended easily to arbitrary planar straight line graphs (PSLGs) within the entire plane, i.e., to a collection of straight-line segments that do not intersect except possibly at common endpoints.
Basic idea

- Simulate the wavefront propagation.
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Computing Straight Skeletons

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Triangulation-Based Algorithm

Aichholzer&Aurenhammer (1998)

- Maintain a kinetic triangulation of (the interior of) the wavefront.
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- Collapsing triangles witness edge and split events.
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Collapsing triangles witness edge and split events.

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Algorithmic insight

Wavefront propagation based on kinetic triangulations allows to determine all events and to compute straight skeletons.
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Offsetting

- How can we determine all offsets that correspond to some user-specified offset distance $t$?
Offsetting

Standard approach

1. Compute an elementary offset segment for each input segment.
Offsetting

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2. Trim at intersections of neighboring segments,
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3. Determine all self-intersections and split into several loops.
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2. Trim at intersections of neighboring segments, and close gaps to form one loop.
3. Determine all self-intersections and split into several loops.
4. Discard excess loops.
Offsetting Based on Straight Skeleton
1. Choose SK edge not yet intersected by an offset loop; compute start vertex.
Offsetting Based on Straight Skeleton

Scan straight skeleton

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2. Advance clockwise along boundary of SK face and compute next vertex.
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3. Move to neighboring face and keep scanning that face clockwise.
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3. Move to neighboring face and keep scanning that face clockwise.
4. Finish one offset curve.
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2. Advance clockwise along boundary of SK face and compute next vertex.
3. Move to neighboring face and keep scanning that face clockwise.
4. Finish one offset curve. Continue with next offset curve.
Offsetting Based on Straight Skeleton

Alternative: Halt wavefront

Halt wavefront-propagation when the offset distance $t$ is reached.
Implementation

Long way to go from the theoretical sketch of Aichholzer&Aurenhammer (1998) to an actual implementation. Need to avoid flip-event loops [Palfrader&Held&Huber (2012)]. Need to handle degeneracies that cause multiple simultaneous events [P .&H.].

SURFER

Straight-skeleton algorithm, based on kinetic triangulations, implemented in C and named SURFER.
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Implementation: Finding and Classifying Collapse Times

Different ways to compute collapse time

Suppose that the three vertices of a triangle move towards one point.
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Suppose that the three vertices of a triangle move towards one point.

- The parabola plotted in blue is the (signed) area of the triangle over time, e.g., as obtained by means of determinant computations.

\[ \Delta \text{ collapses} \]
Implementation: Finding and Classifying Collapse Times

Different ways to compute collapse time

Suppose that the three vertices of a triangle move towards one point.

- The parabola plotted in blue is the (signed) area of the triangle over time, e.g., as obtained by means of determinant computations.
- The function in green represents the (signed) distance of one vertex to its opposite edge.

![Diagram of a triangle with vertices moving towards a point, and graphs of area and distance functions.]
Experiments

- Several straight-skeleton codes:
  - Felkel&Obdržálek (1998),
  - Cacciola/CGAL (2004),
  - Huber&Held (“BONE”, 2010).

SURFER is faster than BONE, which in turn is significantly faster than Cacciola’s CGAL code [Palfrader&Held&Huber (2012)]. No published/non-proprietary codes dedicated to mitered offsetting are known.

CLIPPER and EOS: Polygon-clipping libraries that apply general-purpose Boolean clipping algorithms to compute offsets.
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Experiments

- Simple closed polygons as test data.
- Input complexity $n$ on $x$-axis, running time in seconds on $y$-axis.

Computation of one offset

- CLIPPER,
- SURFER.

Experimental result

SURFER consumes roughly $5.8 \cdot 10^{-7}$ microseconds for an $n$-segment input. Except for a few convex polygons, a full run of SURFER is always (substantially) faster than the computation of one offset by CLIPPER.
Experiments

- Simple closed polygons as test data.
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Computation of one offset

- Clipper,
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SK and SK-based offsetting

- Full SK by Surfer,
- One offset based on SK.

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Surfer consumes roughly $5.8 \cdot 10^{-7} n \log n$ microseconds for an $n$-segment input. Except for a few convex polygons, a full run of Surfer is always (substantially) faster than the computation of one offset by Clipper.
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- Input complexity \( n \) on x-axis, running time in seconds on y-axis.

Computation of one offset

- **CLIPPER**, **SURFER**.

SK and SK-based offsetting

- Full SK by **SURFER**, One offset based on SK.

Experimental result

**SURFER** consumes roughly \( 5.8 \cdot 10^{-7} n \log n \) microseconds for an \( n \)-segment input. Except for a few convex polygons, a full run of **SURFER** is always (substantially) faster than the computation of one offset by **CLIPPER**.
Comparison of Sample Offsets
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Voronoi diagram and rounded offsets
Comparison of Sample Offsets

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Straight skeleton and mitered offsets
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Straight skeleton and mitered offsets

Straight skeleton and beveled offsets
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Linear axis and multi-segment bevels

Straight skeleton and beveled offsets
Thanks for Your Attention — Questions Welcome!

1. Introduction
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   - Change of Wavefront Topology
   - Definition of Straight Skeleton

2. Triangulation-Based Algorithm
   - Basic Idea
   - Kinetic Triangulation

3. Offsetting
   - Standard Approach
   - Offsetting Based on SK

4. Implementation

5. Experimental Results

6. Gallery